A **Quintu***ple 8-Pieces Arrangement Puzzle*

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Recently I in cooperation with two familiar chess problemists from Germany and France have composed a striking unusual puzzle on arrangement of pieces on a chessboard. It was published in the oldest German magazine specializing in chess composition. However, it suits and deserves to be announced to a wider audience.

Arrangement of pieces forms a very popular kind of mathematical chess problems lying far from the game of chess and conventional chess composition (cf. e.g., https://en.wikipedia.org/wiki/Mathematical_chess_problem). Most famous is the 8-queens puzzle posed already in 1848: place eight queens on a chessboard *independently*, that is, in such a way that they do not attack each other (https://en.wikipedia.org/wiki/Eight_queens_puzzle). As Martin Gardner, the author of many remarkable books on recreational mathematics, indicated, there is an enormous literature on this old classic task. Widely known is the answer: there are 92 solutions (12 distinct ones if considered up to rotations and reflections of the chessboard). Another familiar puzzle asks about the minimum number of queens standing independently and *dominating* over all vacant squares, i.e. guarding them. The answer is 5, with plenty of implementations (quite remarkably, 5 queens suffice even on an 11×11-board).

The puzzle under consideration possesses three distinguishing (even unprecedented) features.

(1) First of all, it is a quintuple, i.e. not a single but a set of five closely related and uniformly specified subproblems (called *twins* or, more exactly, *quintuplets*) with distinct solutions.

(2) Further, arrangement problems deal seldom with pawns and fairly seldom with distinct kinds of pieces. On the contrary, we ask to place five sets of chess pieces that consist of all eight ordinary white officers or of one pawn and seven officers.

(3) One and the same simple additional restriction (namely, a preplacement of one piece) is imposed in all cases, and it turns out that each twin has a unique solution. This uniqueness is the most attractive and valuable feature, which is rather atypical for piece arrangements (unlike the popular non-chess game of *Sudoku,* with its small sets of preoccupied cells (“givens”) and the only possible proper completions to be restored).

Let me add that not only composing puzzles like this one but solving them strictly, as well, can only be done computer assisted, what is not unusual now-a-days. However, the solution by itself is quite comprehensible and spectacular.

Quintuple eight-pieces arrangement puzzle (by Valery Liskovets, Werner Keym and Alain Brobecker, *Die Schwalbe*, 2018, H.294-1, #17677).

On a chessboard with the white King standing on f1 *place the following 7 white pieces so that all 8 pieces do not attack each other but guard the rest 56 vacant squares:*

(a) all seven remaining officers, that is, QRRBBNN;

(b) with a Pawn instead of a Knight, the Bishops being of distinct colors;

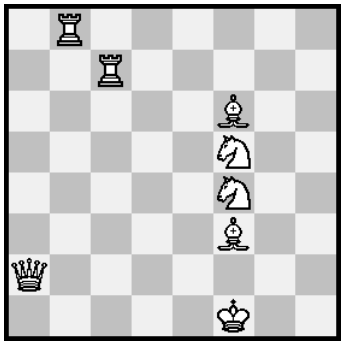
(c) with a Pawn instead of a Knight, the Bishops being of the same color;

(d) with a Pawn instead of the dark-squared Bishop;

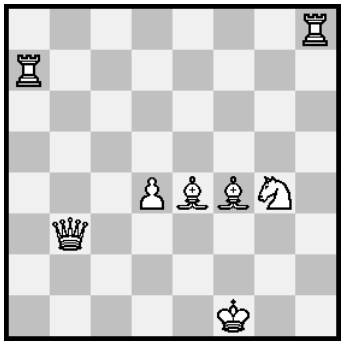
(e) with a Pawn instead of the light-squared Bishop.

Solution. The quintuplets admit the following unique arrangements A – E, resp. (where kB/qB over diagrams denote white kingside/queenside Bishop):

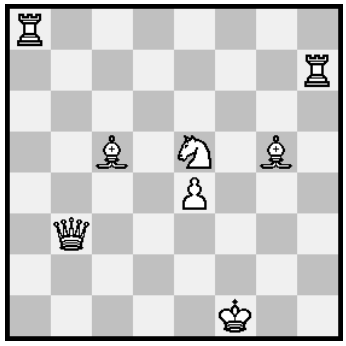
A KQRRBBNN



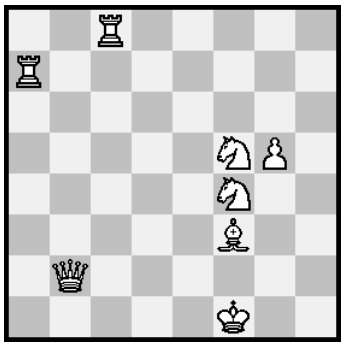
B KQRRkBqBNP



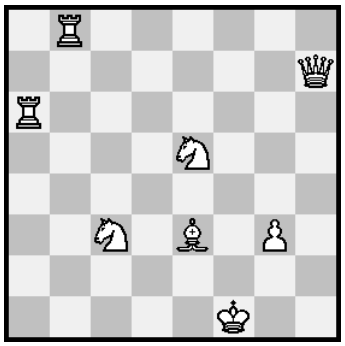
C KQRRB=BNP



D KQRRkBNNP



E KQRRqBNNP



Explanation. These unique solutions have been extracted out of plenty of possible arrangements of the corresponding 8 pieces with the King standing arbitrarily:

(a) out of 176 arrangements (see Appendix);

(b) out of 908;

(c) out of 184 and

(d) and (e) out of 261 arrangements each.

All these arrangements can easily be found with the help of the computer program S*panning.exe* developed by one of the authors (AB) in the connection with this problem (the term "*spanning*" means the independence combined with domination). This convenient program is currently available at http://abrobecker.free.fr/chess/Spanning.zip. An easy case-by-case analysis of its outputs sorted appropriately is required.

Comments.

1. The uniqueness of the solution in every case due to the preplacement of only one and the same piece onto one and the same square is an incredible (even mysterious) phenomenon and lucky find.

2. It is difficult to arrange 8 officers even with the King standing arbitrarily. However, in the cases with a Pawn, constructing several such arrangements is comparably an easy exercise[1].

3. The most impressive contrast takes place in (b) between the single arrangement with the King standing on f1 and 908 arrangements with the King standing arbitrarily: on the average, among the latter, the King occupies every square 14+ times (≈908/64).

4. It is remarkable that the sets of pieces are specified uniformly in terms of the colors of Bishop's squares. Frankly speaking, however, the twin (c) is not fully organic here because of the presence of a "promoted" Bishop (in a rigorous chess-game sense).

5. It is interesting that no more solution would arise even if the Pawn were allowed to stand "illegally" (again in a chess-game sense) on the 1st or 8th rank. E.g., 36+36 new arrangements would appear in (d) and (e) but none with the King standing on f1.

6. I attract the reader's attention to the curious pattern: distinct geometric figures formed by the minor pieces and Pawn. Namely: vertical and horizontal strips in Diagrams A and B, resp., and isosceles triangles lying apex down and up in C and E, resp. (in Diagram D, the figure is unexpectedly similar to that of A but looks less spectacular).

7. Our puzzle does not have direct predecessors. However, it has quite interesting but almost unknown historical roots going back to 1927, when Paul Frey published, for the first time, an independent dominating arrangement of the 8 officers (see the appendix), and even to 1850, when Franz Nauck not only famously posed the 8-queens problem but posed also its subproblem with a preplacement of two queens.

8. One more curious point: diverse roles of the number 8 here and in related problems (cf. the 8-queens puzzle). In particular, 7 officers, or 6 officers plus 1 pawn do not suffice for spanning arrangements, unlike 6 officers plus 2 pawns, e.g., KQRRBNPP.

9. The above-mentioned remarkable history and other details concerning the puzzle, as well as a similar new arrangement problem, can be found in my article entitled (almost identically) "*A quintuple eight-pieces arrangement puzzle*" and written in less popular terms. It was published in the Spanish chess problem magazine *Problemas* (issue 27, July 2019; http://sepa64.blogspot.com.es/p/revista-problemas-nueva-epoca.html).

10. In general, for various arrangement problems (among chess puzzles of other types) I refer the reader to the very interesting website *"The Puzzling Side of Chess"* (http://www.coakleychess.com/puzzlingside) created and supported by the Canadian expert Jeff Coakley, particularly to its columns 15 (*"Eight Officers"*), 100 and 178.

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[1] Here is an outline of a simple implementable way: distinguish a 5×5 sub-board M and place the Queen outside M acting along a diagonal of it; then place the King, three minor pieces and the Pawn inside the board M appropriately with respect to it (a few possibilities); finally, place two Rooks outside M not attacking its squares and each other and not attacked by the rest pieces.

Appendix

Independent dominating arrangements of KQRRBBNN: the full list up to symmetry

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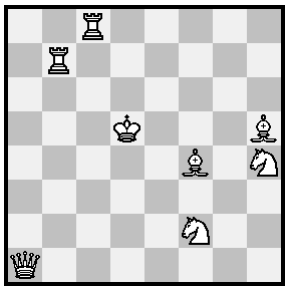

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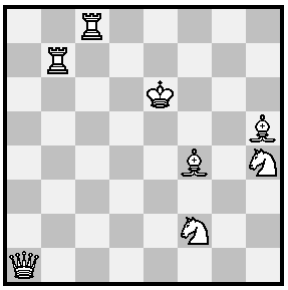

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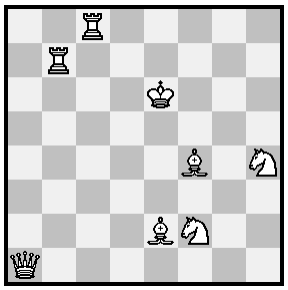

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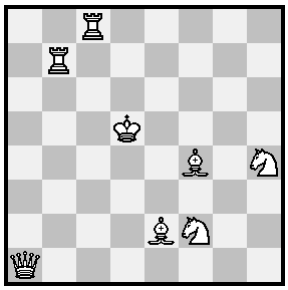

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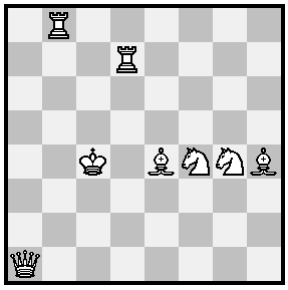

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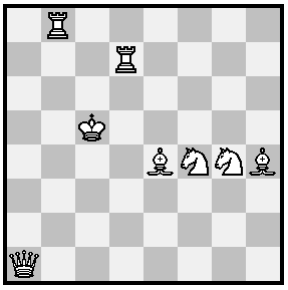

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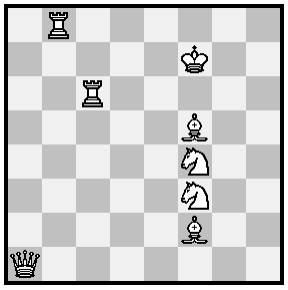

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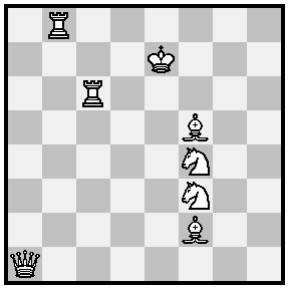

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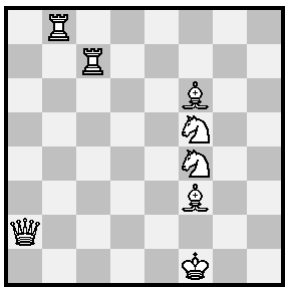

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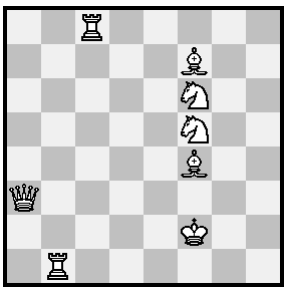

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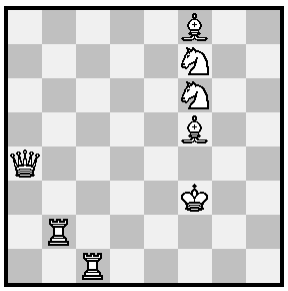

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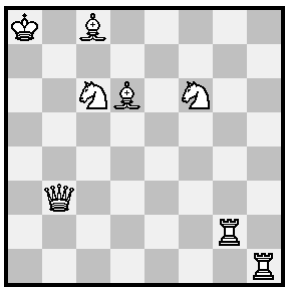

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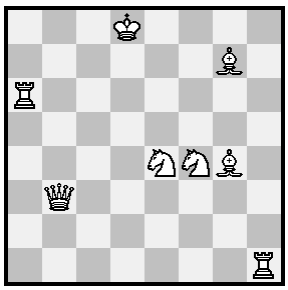

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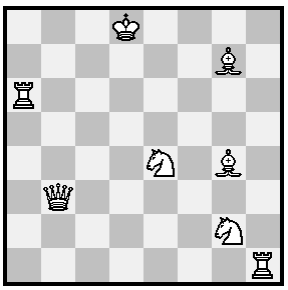

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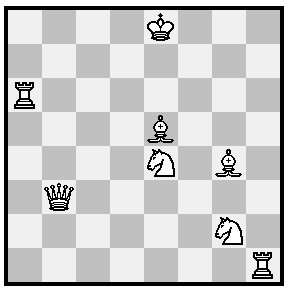

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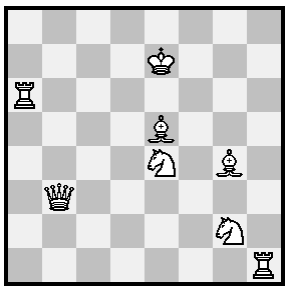

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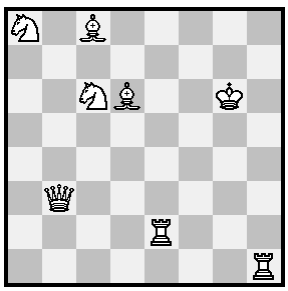

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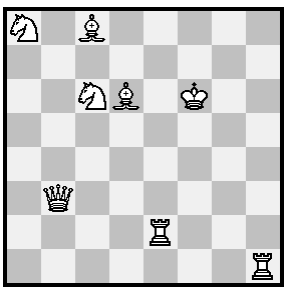

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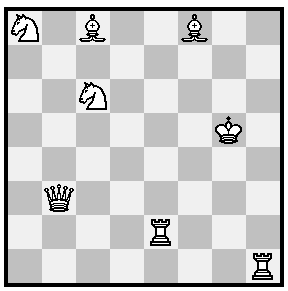

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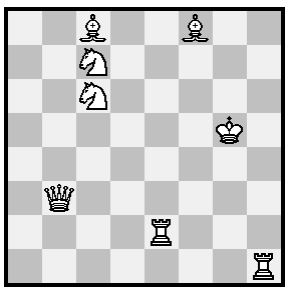

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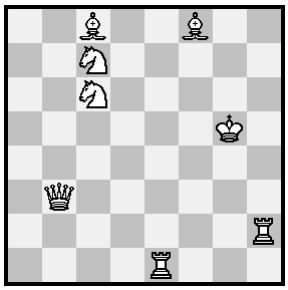

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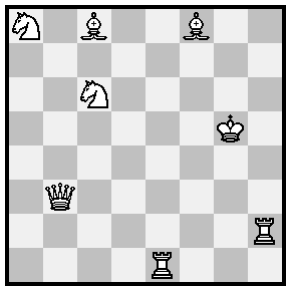

Clarification.

1. Every depicted diagram represents 8 distinct arrangements (an "octet") obtainable from each other by rotations and reflections of the chessboard. Totally: 176 = 22×8 arrangements generated by the program *Spanning.exe*.

2. These particular 22 arrangements have been selected from all 176 ones so that the Queen stands inside the triangle a1–a4–d4–a1 (a one eighth slice); moreover, for the mirror-symmetric place Qa1, the King has been chosen to stand above the diagonal a1–h8. Every octet contains exactly one such arrangement.

3. Rather unexpectedly, all arrangements turn out to have distinct-colored bishops.

4. P.Frey in 1926–1933 discovered six independent dominating arrangements of the eight officers, namely, those represented by Diagrams 7, 8, 9, 10, 11 and 12. Among them, No.12 is the earliest arrangement found by him and No.9 is our Diagram A. The other 16 arrangements seem to have been unknown.

5. In no diagram other than No.9 the King occupies f1 or one of the squares c1, a3, a6, etc. corresponding to f1 by symmetry. Thus, we convince ourselves that among all 176 arrangements only the depicted one contains Kf1, just as we need.

6. Statistics with respect to the place of the Queen:

22 = 8(Qa1) + 1(Qa2) + 1(Qa3) + 1(Qa4) + 11(Qb3);

176 = 4(corners)×2×8 + 24(edge)×1 + 8(b3, b6, c2, etc.)×11 + 28(the rest squares)×0.

7. One can clearly partition the arrangements into 5 similarity clusters as follows:

{1,2,3,4} – six pieces are fixed while the place of the King or one Bishop varies among these diagrams;

{5,6,7,8,9,10,11} – all contain a "strip" of four light pieces BNNB and also K standing on the same line or nearby (for a greater visual uniformity, Diagrams 5 and 6 could be turned (a1→h1) while Diagrams 7 and 8 be reflected (a1↔a8));

{12} – a sole, exclusive arrangement (no similar);

{13,14,15,16} – five pieces are fixed while the places of K, one B and one N vary;

{17,18,19,20,21,22} – three pieces are fixed while the places of one or two pieces out of K, one B and one N or of RR (minimal shifts) vary among these diagrams.

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(A Quintuple 8-Pieces Puzzle.pdf, valery.liskovets@gmail.com)