Maximum volume for a Moda Masu Square Box

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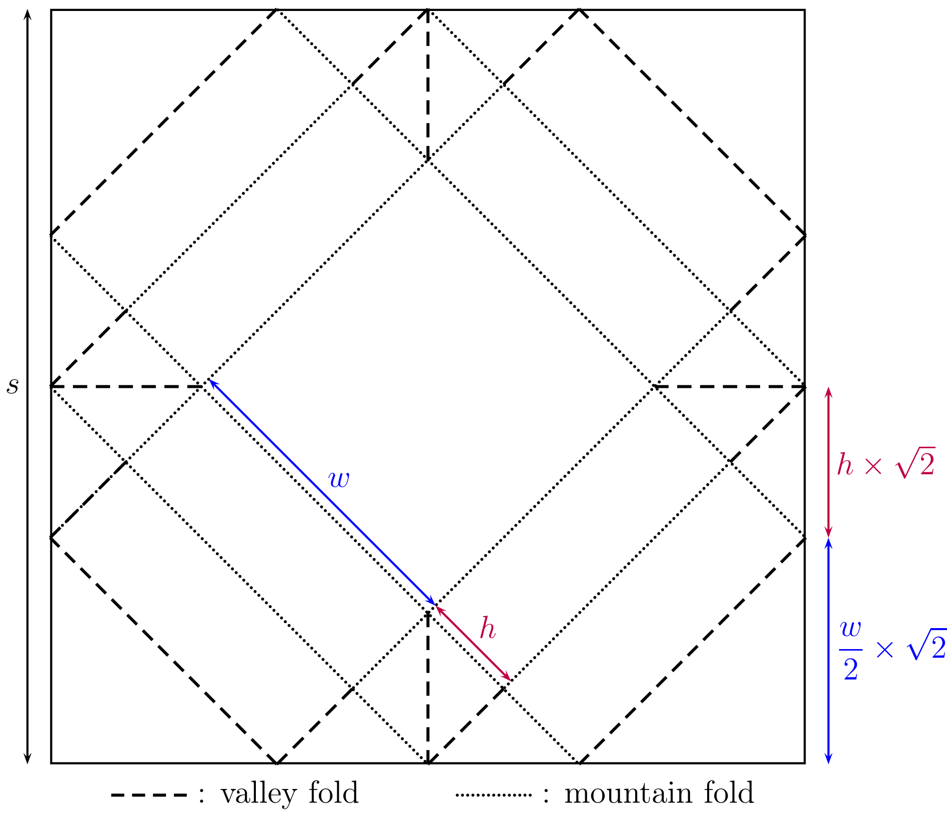
JavaScript program to create Moda Masu Square Boxes

A Moda Masu Square Box can be obtained by folding the pattern shown below:



Given a square sheet of paper of side length s, how can we construct a Moda Masu Box with a square base from this sheet so that its volume V is maximized?

If we call w the side of the square base of the box and h the height of the box, then we have:

$$w \in \left]0; \frac{s \times \sqrt{2}}{2}\right[\quad \text{and} \quad h \in \left]0; \frac{s \times \sqrt{2}}{4}\right[$$

$$V = w^2 \times h$$

$$s = 2 \times \left(\frac{w}{2} \times \sqrt{2} + h \times \sqrt{2}\right)$$

From the formula of s we find:

$$h = \frac{s \times \sqrt{2} - 2 \times w}{4}$$

Which, when replaced in V gives:

$$V(w) = \frac{w^2 \times s \times \sqrt{2} - 2 \times w^3}{4}$$

Let us study the polynomial function $V(w)$ to find the value w where it reaches its maximum value.

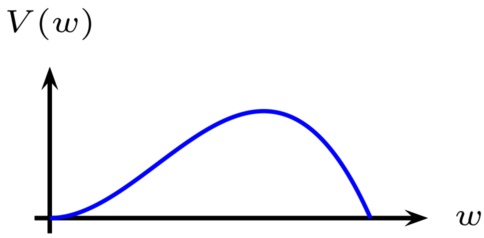
$$V'(w) = \frac{w}{2} \times \left(s \times \sqrt{2} - 3 \times w\right)$$

$$V'(w) = 0 \iff w = 0 \quad \textit{or} \quad s \times \sqrt{2} - 3 \times w = 0$$

$$\iff w = 0 \quad \textit{or} \quad w = \frac{s \times \sqrt{2}}{3}$$

Below are the function variation table of V(w) and a plot of the function:

w	0	$+$	$\frac{s\times\sqrt{2}}{3}$	$+$	$\frac{s\times\sqrt{2}}{2}$
$s\times\sqrt{2}-3\times w$		$-$	0	$+$	
$V'(w)$		$-$	0	$+$	
$V(w)$	0	↗	$\frac{s^3\sqrt{2}}{54}$	↘	0



So the maximum is reached for $w_M = \frac{s\times\sqrt{2}}{3}$ and the volume will be $V\left(\frac{s\times\sqrt{2}}{3}\right) = \frac{s^3\sqrt{2}}{54}$.

More interestingly $h_M = \frac{s\times\sqrt{2} - 2\times\frac{s\times\sqrt{2}}{3}}{4} = \frac{s\times\sqrt{2}}{12}$.

And this says that $\boxed{h_M = \frac{1}{4}\times w_M}$ gives the maximum volume.

Last, this also means that in order to have a maximum volume, you must divide each side of the sheet according to the following subdivisions: