Polyssimo Challenge - Retrograde Analysis

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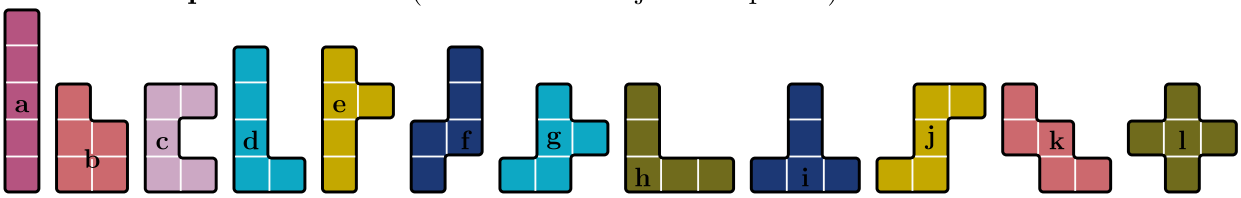
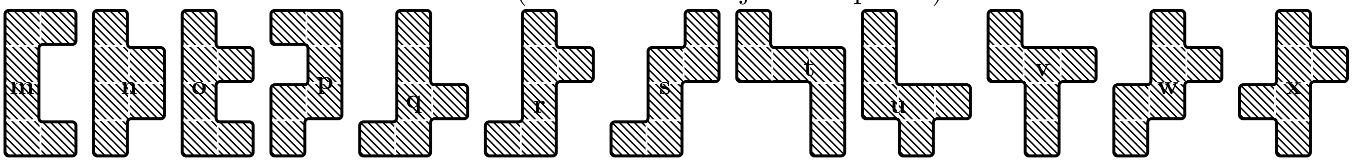
I'm a big fan of **retrograde analysis** problems (in chess and other games), basically the idea is to give the position of a game, and you must discover what was played or part of it. Instead of finding the best move, you go backward in time. After quite a long time, I was able to make a retrograde analysis problem in my own game **Polyssimo Challenge**. Before showing it to you, maybe it's a good idea to present you the ...

RULES : Polyssimo Challenge is a tactical game for 2 to 4 players. It consists of a board with 12×11 squares and 24 reversible pieces :

• 12 colored "**pentominoes**" (made with 5 adjacent squares) :



• 12 black and white "**hexominoes**" (made with 6 adjacent squares) :



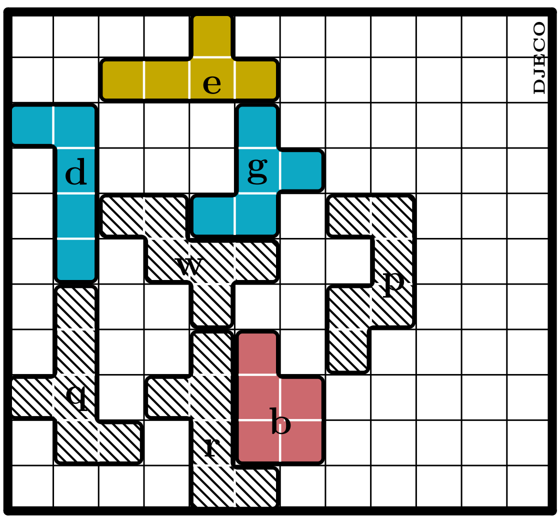
At first, the players choose their pieces in turn and put them in their reserve.

When all the pieces have been taken, the players, beginning with the last one who took a piece and by reversing the order of play (which compensates the disadvantage of having chosen last), strive to place their own pieces on the board.

If a player can not place a piece, he passes his turn.

When no one can place a piece anymore, each player counts the number of squares of his unplaced pieces, the winner is the one with the fewest squares. In case of equality, the player who has placed the last piece wins.

Below is a first retrograde analysis problem for Polyssimo Challenge, a **proof game** in 4.0 moves (or 8 plies). I had to tweak a bit the pieces attribution to allow for such a problem, but maybe it's possible without or to have an even longer problem ?



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In this strange Polyssimo Challenge game, one player has taken all pentominoes when the other player has taken all hexominoes. Above is the board after 8 plies, and we know that nothing happened on the first two plies, but on subsequent plies each of the player has created one space in which only him can place some of his remaining pieces (such space is called a reserved area or a footprint). Who started and how did the game went ?

Below is a problem made by the chess retrograde analysis World Champion 2003 **Thierry Le Gleuher** (2020/10/31), in which you have to discover the pieces taken by the players :

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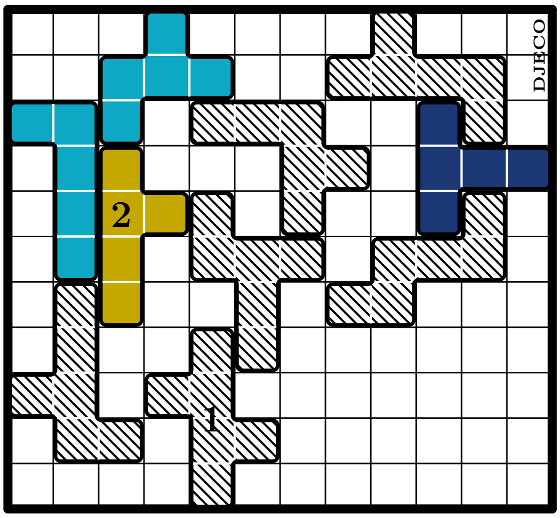

In this Polyssimo Challenge game the first two pieces played on board are indicated, and after this two plies each player has created one space in which only him can place some of his remaining pieces (such space is called a reserved area or a footprint). What pieces were taken by the first player ?

Another problem by **Thierry Le Gleuher** (2020/11/04) :

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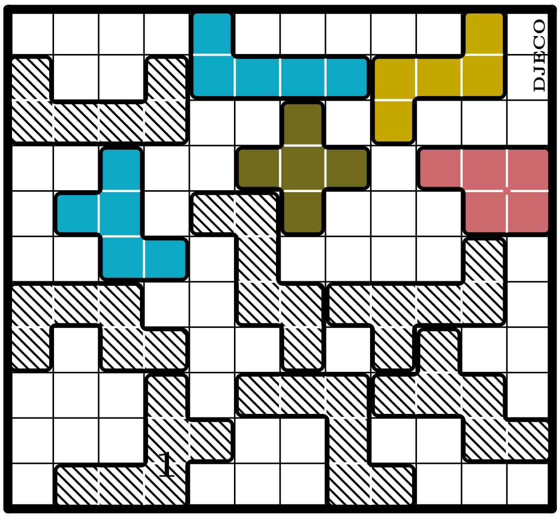
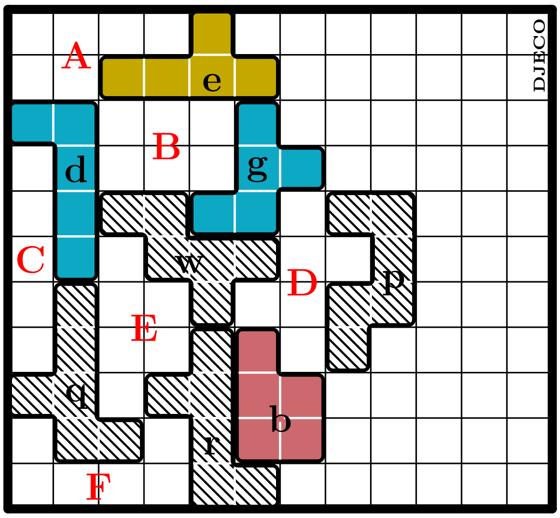


In this Polyssimo Challenge game the first move played on board is indicated. From the third ply and on, each player has created one space in which only him can place some of his remaining pieces (such space is called a reserved area or a footprint). How did the game went ?

Solutions



1

A, **B**, **C** are reserved areas for $\mathcal{P}$, the player owning the $\mathcal{P}$entominoes.
D, **E**, **F** are reserved areas for $\mathcal{H}$, the player owning the $\mathcal{H}$exominoes.
Pieces **e** and **g** must have been played before **D** is created to have it as a reserved area for $\mathcal{H}$.
Pieces **b** and **e** must have been played before **E** is created to have it as a reserved area for $\mathcal{H}$.
Piece **d** must have been played before **F** is created to have it as a reserved area for $\mathcal{H}$.

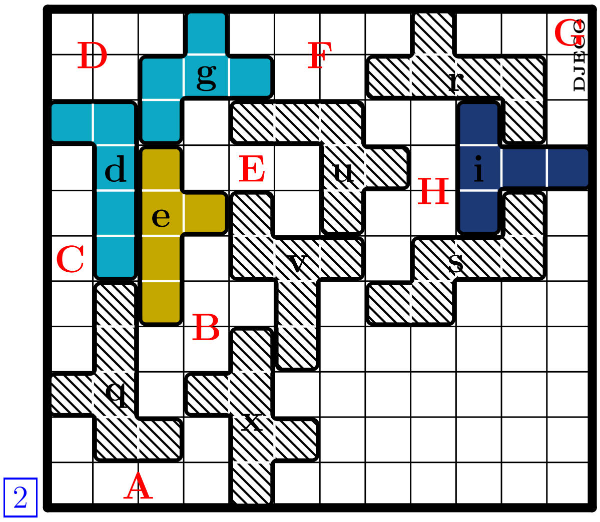
F is the first reserved area created by $\mathcal{H}$ because **E** needs 3 b&w pieces so cannot appear before 5^{th} ply and **D** would need pieces **b**, **g**, **p** and **w** to be played in plies 1 to 4, which creates no other reserved area. Piece **d** needs to be played before **F** is closed to consider this as a reserved area for $\mathcal{H}$, and since **d** is the piece placed by $\mathcal{P}$ to create the footprint **C** we conclude that **C** was created before **F** and so we have :

Ply 1 : $\mathcal{P}$ plays **?**
Ply 2 : $\mathcal{H}$ plays **q**
Ply 3 : $\mathcal{P}$ plays **d** and creates **C**
Ply 4 : $\mathcal{H}$ plays **r** and creates **F**

From this we deduce that $\mathcal{H}$ will play **w** on 6^{th} ply to create **E** and **p** on 8^{th} ply to create **D**.
We also know that $\mathcal{P}$ created one footprint with **e** and one footprint with **g**, so they were played in this order on 5^{th} and 7^{th} plies (the opposite order would create no footprints when placing **g** and two footprints when placing **e**). This is also consistent with the order required so that $\mathcal{H}$ creates his own reserved areas. Only **b** remains and was played on 1^{st} ply. The full game is then :

Ply 1 : $\mathcal{P}$ plays **b**
Ply 2 : $\mathcal{H}$ plays **q**
Ply 3 : $\mathcal{P}$ plays **d** and creates **C**
Ply 4 : $\mathcal{H}$ plays **r** and creates **F**
Ply 5 : $\mathcal{P}$ plays **e** and creates **A**
Ply 6 : $\mathcal{H}$ plays **w** and creates **E**
Ply 7 : $\mathcal{P}$ plays **g** and creates **B**
Ply 8 : $\mathcal{H}$ plays **p** and creates **D**

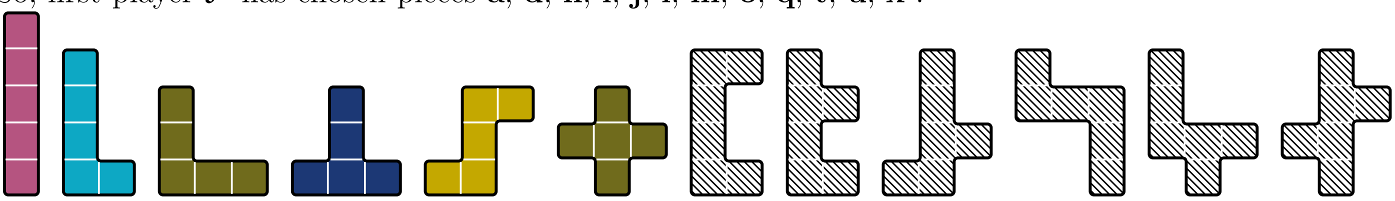
It is easy to add one ply to that proofgame, eg by using **k** to create a footprint for **i** or by using **j** to create a reserved area for **h**, but I decided against it because it would immediatly say who started and I felt it was less elegant. So I bet the next step is to have a proofgame in 10 plies or without the Pentominoes/Hexominoes separation (which I searched in vain up to now, except for very short games).

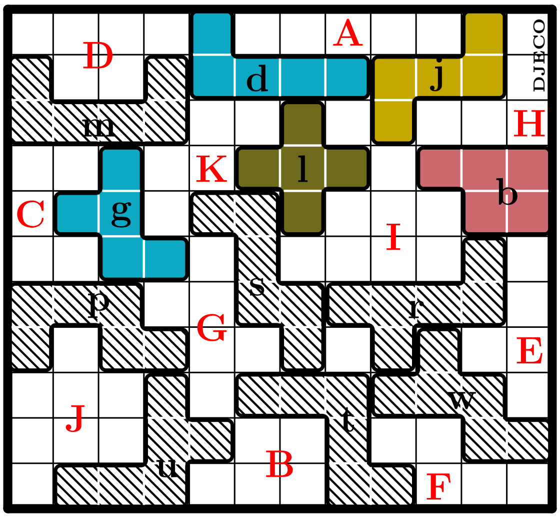


On ply 3, after pieces **x** and **e** are played, only placing piece **q** allows to create reserved area **A** which can contain pieces **d** or **m**. Thus this two pieces are owned by first player $\mathcal{F}$.
Since the second player $\mathcal{S}$ does not own piece **d**, his only possibility to make a reserved area is playing **v** and creating reserved area **B** which can contain pieces **f**, **k**, **g** and **w**. Thus this four pieces are owned by second player.
Continuing like this we find :

Ply 1 : $\mathcal{F}$ plays **x**
Ply 2 : $\mathcal{S}$ plays **e**
Ply 3 : $\mathcal{F}$ plays **q** and creates **A** $\Rightarrow$ $\mathcal{F}$ owns **d** and **m**
Ply 4 : $\mathcal{S}$ plays **v** and creates **B** $\Rightarrow$ $\mathcal{S}$ owns **f**, **k**, **g** and **w**
Ply 5 : $\mathcal{F}$ plays **d** and creates **C** $\Rightarrow$ $\mathcal{F}$ owns **a**
Ply 6 : $\mathcal{S}$ plays **g** and creates **D** $\Rightarrow$ $\mathcal{S}$ owns **b**
Ply 7 : $\mathcal{F}$ plays **u** and creates **E** $\Rightarrow$ $\mathcal{F}$ owns **j**
Ply 8 : $\mathcal{S}$ plays **r** and creates **F** $\Rightarrow$ $\mathcal{S}$ owns **n**
Ply 9 : $\mathcal{F}$ plays **i** and creates **G** $\Rightarrow$ $\mathcal{F}$ owns **h**
Ply 10 : $\mathcal{S}$ plays **s** and creates **H** $\Rightarrow$ $\mathcal{S}$ owns **c**, **f** and **p**

Summing up, we see that first player $\mathcal{F}$ owned **x**, **q**, **d**, **u**, **i**, **m**, **a**, **j**, **h** which gives only 9 pieces he has chosen...
But as is usual with Thierry's problem, we simply look elsewhere to have our answer : the second player $\mathcal{S}$ owned **e**, **v**, **g**, **r**, **s**, **f**, **k**, **w**, **b**, **n**, **c**, **p** which is 12 pieces, thus all pieces not owned by him were owned by first player $\mathcal{F}$.
So, first player $\mathcal{F}$ has chosen pieces **a**, **d**, **h**, **i**, **j**, **l**, **m**, **o**, **q**, **t**, **u**, **x** :





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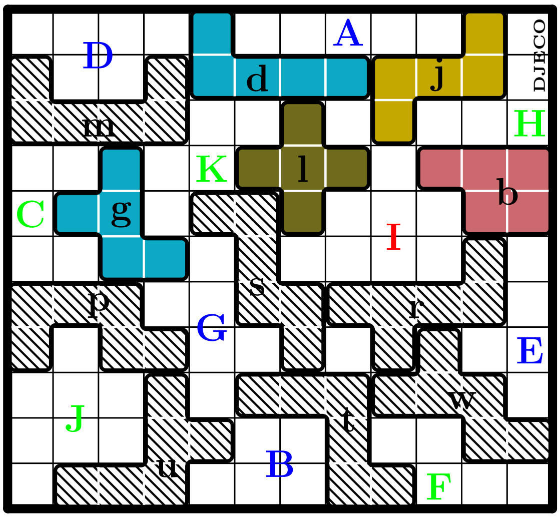

We can see 11 existing zones, but from the stipulation only 10 reserved areas were created.
A, **C**, **E**, **F**, **H** and **K** are perfect footprints for the pieces **a**, **c**, **e**, **f**, **h** and **k** respectively.
B is a perfect footprint for the piece **b**, but this one has already been played (hence the box around it).
D can contain pieces **e**, **n** and **b**.
G can contain pieces **e**, **x** and **g**.
J can contain pieces **c**, **f**, **i**, **k**, **h**, **v** and the already placed **b**, **g**, **j**, **l**, **u**, **w**.
I can contain pieces **c**, **e**, **f**, **h**, **i**, **k**, **n**, **o** and the already placed **b**, **d**, **g**, **l**, **p**, **u**, **w**.

Since **I** can contain 8 of the remaining pieces, we conclude that it's not a reserved area. Hence all other areas are reserved areas!
From this we deduce one player, let's call him $\mathcal{G}$reen, still owns the pieces **c**, **f**, **i**, **k**, **h**, **v** that are fitting in the reserved area **J**, and this player also created the areas **C**, **F**, **H** and **K** which makes 5 reserved areas. We then know the other player, let's call him $\mathcal{B}$lue, owns the pieces **a**, **e**, **n**, **o**, **q**, **x** and he created the reserved areas **A**, **B**, **D**, **E**, and **G**. We also know he owned the **b** piece and played it after the reserved area **B** was created, so it cannot have been the second placed piece.

Now, let's consider the last piece played : It must have closed only one reserved area, and only two pieces satisfy this condition, namely **l** and **r**. All other pieces would close two reserved areas.

Since **b** is a piece owned by $\mathcal{B}$ and it cannot be the second placed piece, then it must have closed a reserved area owned by $\mathcal{B}$, and the only possibility is that it has closed **E**. From this we conclude that **r** cannot be the last played piece closing **E**. Hence the last played piece is **l**.

From then, we retract the moves taking in account that we must alternate closure of blue and green reserved areas (and only one per move). The only possible retracted game is :

Ply 1 : $\mathcal{B}$ plays **u**
Ply 2 : $\mathcal{G}$ plays **r**
Ply 3 : $\mathcal{B}$ plays **t** and creates **B**
Ply 4 : $\mathcal{G}$ plays **w** and creates **F**
Ply 5 : $\mathcal{B}$ plays **b** and creates **E**
Ply 6 : $\mathcal{G}$ plays **j** and creates **H**
Ply 7 : $\mathcal{B}$ plays **d** and creates **A**
Ply 8 : $\mathcal{G}$ plays **p** and creates **J**
Ply 9 : $\mathcal{B}$ plays **m** and creates **D**
Ply 10 : $\mathcal{G}$ plays **g** and creates **C**
Ply 11 : $\mathcal{B}$ plays **s** and creates **G**
Ply 12 : $\mathcal{G}$ plays **l** and creates **K**