Rotations to align an axial cube along its main diagonal

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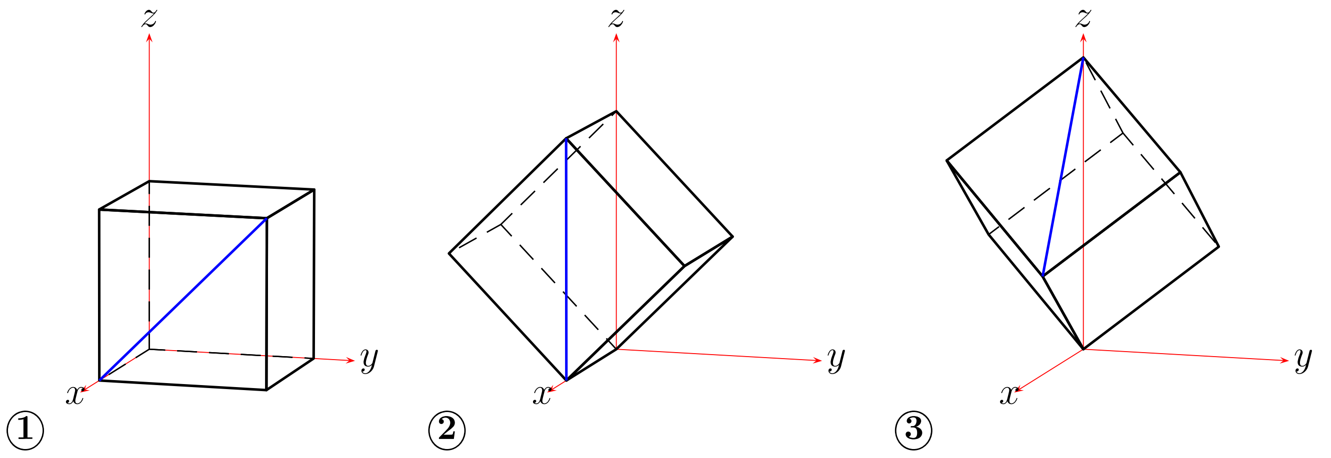
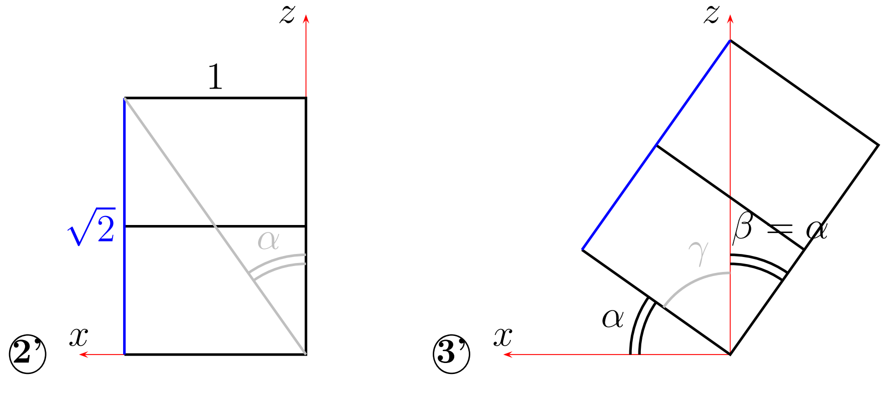


Suppose that we have a cube with an edge of size 1 unit and with vertices aligned along the x, y and z axis. According to Pythagora's theorem the blue diagonal of the "front" face has size $\sqrt{2}$.
Between ① and ② we perform a 45° rotation along the x axis.
Looking at the cube along the y axis we have picture ②' showing a rectangle of edges 1 and $\sqrt{2}$. We now search which rotation of angle α around the y axis will allow to have the main diagonal of the cube aligned around the z axis as in figure ③ and ③'.



To find this angle α, an easy way is to notice first using picture ③' that it is equal to the angle β between the rightmost edge of the rectangle and the z axis: $\alpha + \gamma = 90°$ (because the axis are perpendicular) and $\gamma + \beta = 90°$ (because we have a rectangle) thus $\alpha = \beta$.

Then using the trigonometry formulaes in a right triangle, we have:

$$\tan(\alpha) = \frac{opposite}{adjacent} = \frac{1}{\sqrt{2}}$$

$$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \simeq 35.26438968° \qquad (0.61547970867\ rad)$$

Thus, to align the main diagonal of the cube with the z axis you have to rotate around the x axis by 45°, then rotate around the y axis by -35.264°.