Search Longuest Orthogonal Path

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I recently discovered the tower defense game by OldJ (http://oldj.net/static/html5-tower-defense-en/td.html). I've never played those kind of games before and my interest was great for it. I thought it was deterministic and that an aim was to find the correct positionning of towers so that the path was the longuest possible.
Sadly the game was not deterministic, and exactly the same setup can lead to different scores.

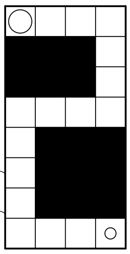
Anyway i took interest in searching the longuest orthogonal path in a $M \times N$ grid, from one corner to the opposite one (corners not counted in the length). The results are summarised in the following table, which gives the length of longuest path in a $M \times N$ grid:

	2	3	4	5	6	7	8	9	10	11
2	1*	2	3	6!	7	8	9	12!	13	14
3		3	4	9!	10	11	12	17!	18	19
4			5	12!	13	14	15	22!	23	24
5				15*	18	21	24	27	30	33
6					21	26!	29	32	37	40
7						29	34	37	42	47
8							37	44	47	52
9								49	54	59
10									61*	66
11										$\geqslant 73$

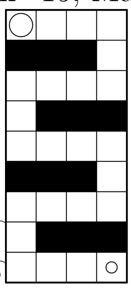
!: unique path
*: unique path, up to the symmetries

First number is solution nb, second is length, third is amount of walls on the border

4 × 8 MaxLength=15, MaxWalls=10, MinWalls=4

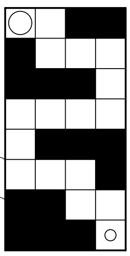


65,15,5



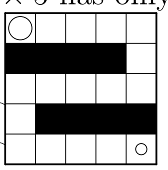
0,22,4

4 × 9 has only one solution, MaxLength=22, MaxWalls=MinWalls=4



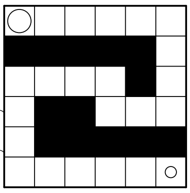
121,15,10

5 × 5 has only one solution, up to symmetries, MaxLength=15, MaxWalls=MinWalls=2

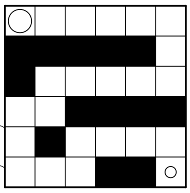


0,15,2

6 × 6, MaxLength=21, MaxWalls=5, MinWalls=2

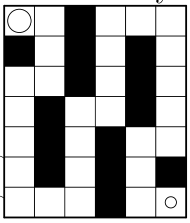


14,21,2



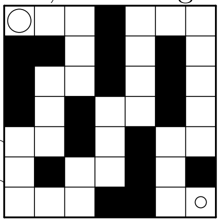
5,21,5

6×7 has only one solution, MaxLength=26, MaxWalls=MinWalls=4

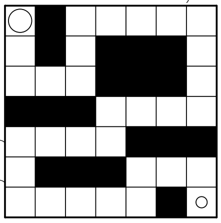


0,26,4

7×7, MaxLength=29, MaxWalls=7, MinWalls=4

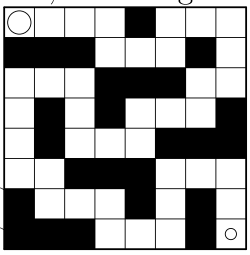


10,29,7

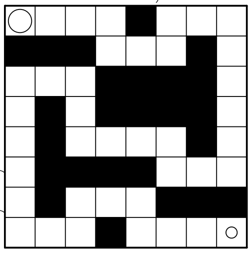


41,29,4

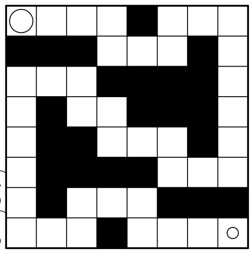
8×8, MaxLength=37, MaxWalls=9, MinWalls=4



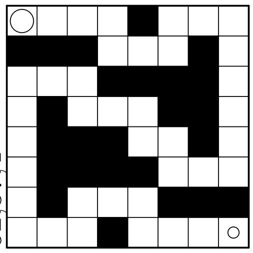
0,37,9



50,37,4

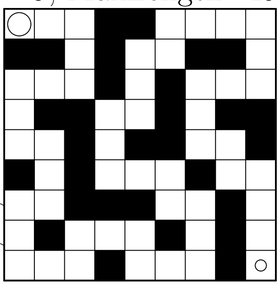


51,37,4

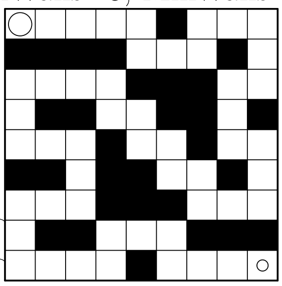


52,37,4

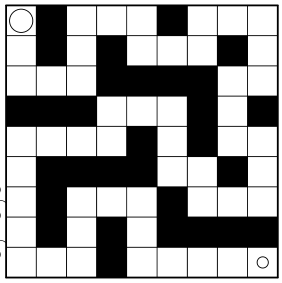
9×9, MaxLength=49, MaxWalls=8, MinWalls=6



11,49,8

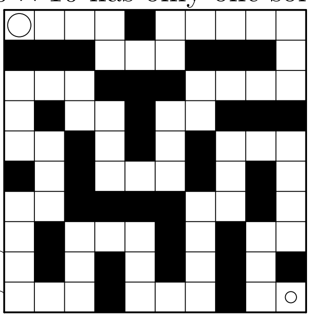


1,49,6



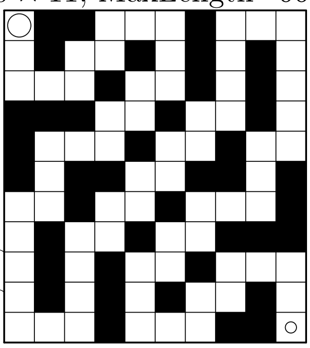
15,49,6

10×10 has only one solution, up to symmetries, MaxLength=61, MaxWalls=MinWalls=7

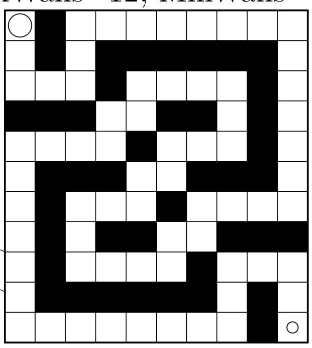


0,61,7

10×11, MaxLength=66, MaxWalls=12, MinWalls=4



863,66,12

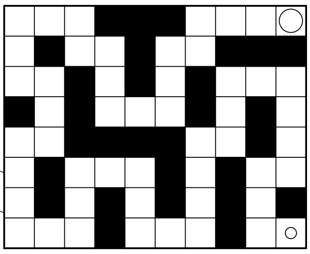


958,66,4

On remarque que le 10×10 peut être trouvé à partir de 10×8 avec l'entrée et la sortie sur la même verticale. Ci dessous on a une longueur de 48 à laquelle on rajoute 13 pour obtenir le 10×10 à l'aide d'un chemin amenant du coin en haut à gauche du 10×10 vers le coin en haut à droite du 10×8 Lorsqu'on connait la longueur max d'un $m \times n$ vertical, cette méthode permet d'obtenir un minorant du $m \times (n+2)$, et on regarde quand il y a au moins un groupe de 3 murs sur le bord haut du $m \times n$.

10×8 vertical, MaxLength=48, MaxWalls=8, MinWalls=4

74,48,8



→

0,61,7