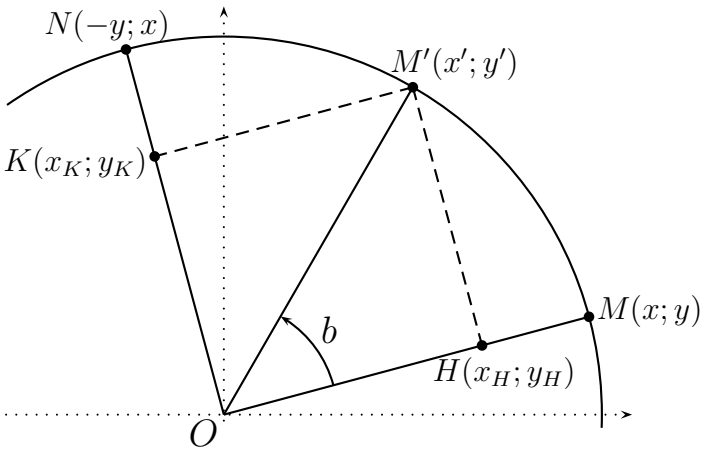
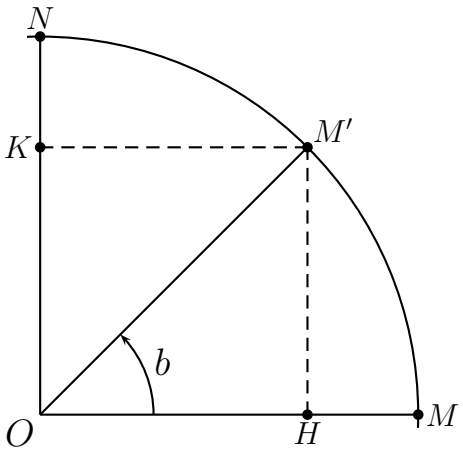
Formules d'addition:

$$cos(a+b) = cos(a)cos(b) - sin(a)sin(b)$$
$$cos(a-b) = cos(a)cos(b) + sin(a)sin(b)$$
$$sin(a+b) = sin(a)cos(b) + cos(a)sin(b)$$
$$sin(a-b) = sin(a)cos(b) - cos(a)sin(b)$$

Démonstration:

① Formules pour une rotation d'angle b autour du point O:





Le rapport $\frac{OH}{OM}$ vaut $\frac{cos(b)}{1}$ et le rapport $\frac{OK}{ON}$ vaut $\frac{sin(b)}{1}$, donc:

$$\begin{cases} x_H = x \times cos(b) \\ y_H = y \times cos(b) \end{cases} \text{ et } \begin{cases} x_K = -y \times sin(b) \\ y_K = x \times sin(b) \end{cases}$$

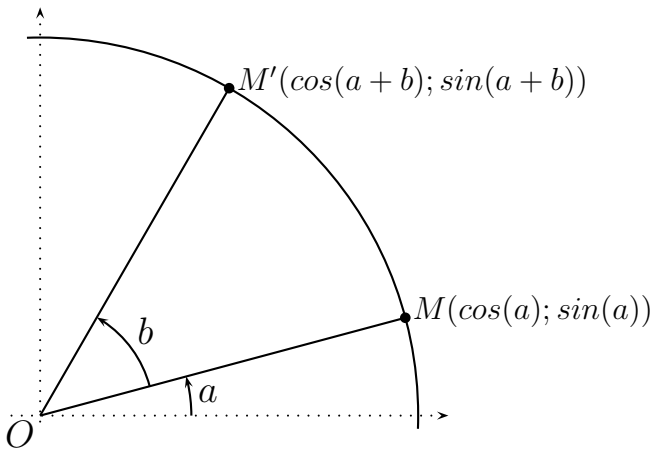
Comme $\overrightarrow{OM'} = \overrightarrow{OH} + \overrightarrow{HM'} = \overrightarrow{OH} + \overrightarrow{OK}$ alors on obtient:

$$\begin{cases} x' = x_H + x_K \\ y' = y_H + y_K \end{cases}$$

et finalement:

$$\boxed{\begin{cases} x' = x \times cos(b) - y \times sin(b) \\ y' = y \times cos(b) + x \times sin(b) \end{cases}}$$

② Application au calcul de $cos(a+b)$:



D'après les formules précédentes:

$$\begin{cases} x' = cos(a+b) = x \times cos(b) - y \times sin(b) = cos(a) \times cos(b) - sin(a) \times sin(b) \\ y' = sin(a+b) = y \times cos(b) + x \times sin(b) = sin(a) \times cos(b) + cos(a) \times sin(b) \end{cases}$$